



Two Factor ANOVA



Greg C Elvers

1

Factorial Designs

- ⊞ Often researchers want to study the effects of two or more independent variables at the same time
 - ⊞ Does it matter where a list of words is studied, on the beach or under water?
 - ⊞ Does it matter where a list of words is recalled, on the beach or under water?

2

Factorial Designs

- ⊞ *Factor* is another name for *independent variable*
 - ⊞ The preceding example has two factors: where you study and where you recall
- ⊞ In a *factorial design*, all possible combinations of the factors are present

3

A Factorial Design

		Study	
		On Beach	Underwater
Recall	On Beach	Study on beach; recall on beach	Study underwater; recall on beach
	Underwater	Study on beach; recall underwater	Study underwater; recall underwater

4

Naming Factorial Designs

- Factorial designs are referred to by the number the number of IVs and the number of levels of each IV
- A design with two IVs is said to be an $n \times m$ (read n by m) design
 - n is replaced with the number of levels, or conditions, of the first IV
 - m is replaced with the number of levels, or conditions, of the second IV

5

Naming Factorial Designs

- The preceding example is a 2×2 factorial design because there are two IVs, and each IV has two levels
- What would you call a design that had 3 IVs in which the first IV had 2 levels, the second IV had 3 levels and the third IV had 4 levels?
- $2 \times 3 \times 4$

6

Factorial Designs

- The number of conditions in a factorial design is equal to the product given by its name

Design	# Conditions
2×2	4
$2 \times 3 \times 4$	24
3×4	12

7

Information From A Factorial Design

- An $n \times m$ factorial designs is very powerful because it allows us to answer three questions:
 - Is there an effect of the first IV?
 - Do you recall more words when you study them on the beach or underwater?
 - Is there an effect of the second IV?
 - Do you recall more words when you recall them on the beach or underwater?

8

Information From A Factorial Design

- ⊞ An $n \times m$ factorial design is very powerful because it allows us to answer three questions:
 - ⊞ Are the effects of the two IVs independent of each other?
 - ⊞ When recalling on the beach, does it matter whether you studied underwater or not? When recalling underwater, does it matter whether you studied underwater or not?

9

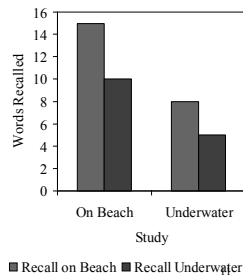
Main Effects

- ⊞ Each of the first two questions (Is there an effect of the first / second IV?) is asking whether there is a *main effect* of that IV
 - ⊞ A main effect occurs when an independent variable has an influence on the dependent variable
 - ⊞ If people recalled more words when they studied them on the beach than when they studied them underwater, then there would be a main effect of where the words are studied

10

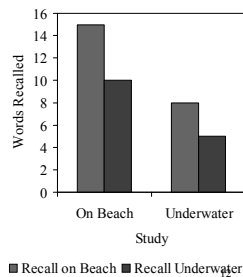
Main Effects

- ⊞ When looking at the main effect of one IV, you should ignore the existence of the other IV
 - ⊞ Compare all conditions that have one level of the IV to all conditions that have the other level of the IV



Main Effects

- ⊞ For the main effect of where the words were studied:
 - ⊞ Average the values of the left two bars (conditions in which people studied on the beach)
 - ⊞ $(15 + 10) / 2 = 12.5$
 - ⊞ Average the values of the right two bars (conditions in which people studied underwater)
 - ⊞ $(8 + 5) / 2 = 6.5$



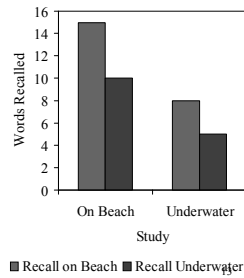
Main Effects

- ⊞ Compare the two means

- ⊞ $\bar{X}_{\text{beach}} = 12.5$ vs $\bar{X}_{\text{underwater}} = 6.5$

- ⊞ The larger the difference between the means, the more likely there is a main effect of the independent variable

- ⊞ There probably is a main effect of where the person studied the words

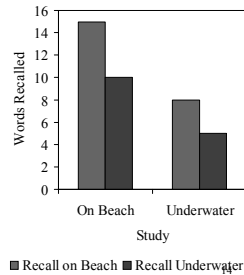


Main Effects

- ⊞ Is there probably a main effect of where the person recalled the words?

- ⊞ Average the values of the two green bars (conditions in which the words were recalled on the beach)

- ⊞ $(15 + 8) / 2 = 11.5$



Main Effects

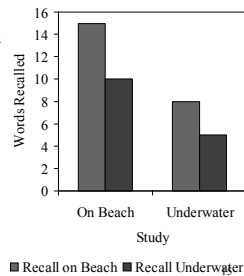
- ⊞ Average the values of the two blue bars (conditions in which the words were recalled underwater)

- ⊞ $(10 + 5) / 2 = 7.5$

- ⊞ Compare the two averages

- ⊞ 11.5 vs 7.5

- ⊞ There probably is a main effect of where the words are recalled



Main Effects

- ⊞ Main effects can be determined from both single factor experiments and from factorial design experiments

- ⊞ However, factorial designs are more efficient than single factor experiments because you can answer the same questions with fewer participants

Interactions

- ⊞ The third question that we can ask about a factorial design is:
 - ⊞ Are the effects of the two IVs independent of each other?
- ⊞ This type of effect is called an *interaction effect* or just an *interaction*
- ⊞ Interactions are very important in research and are the real reason why factorial designs are performed

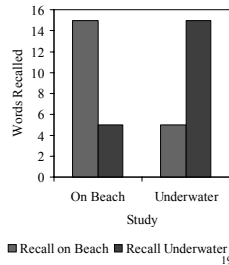
17

Definitions of *Interaction*

- ⊞ There are several equivalent ways of defining *interaction*:
 - ⊞ An interaction occurs when the nature of the simple main effect of one IV depend on then level of the other IV
 - ⊞ An interaction occurs when the effects of one IV cannot simply be added to the effects of the other IV in order to predict how both treatments will simultaneously affect the DV
 - ⊞ An interaction occurs when the lines on a graph of the results are not parallel

Simple Main Effect Definition of *Interaction*

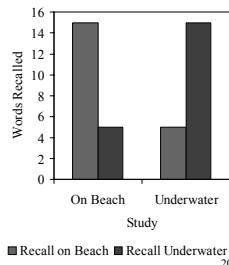
- ⊞ An interaction occurs when the nature of the simple main effect of one IV depend on then level of the other IV
- ⊞ Does the effect of whether you study dry or wet depend on whether you recall dry or wet?



19

Simple Main Effect Definition of *Interaction*

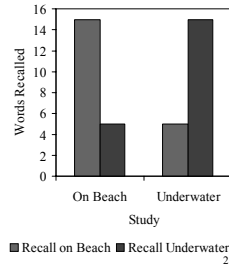
- ⊞ Yes
- ⊞ If you are recalling on the beach, then studying on the beach is better than studying underwater
- ⊞ If you are recalling underwater, then studying on the beach is poorer than studying underwater



20

Simple Main Effect Definition of *Interaction*

- ⊞ The simple main effect of whether you do better studying on the beach or underwater depends on whether your recall on the beach or underwater
- ⊞ Thus, the two variables (where you study and where you recall) interact



21

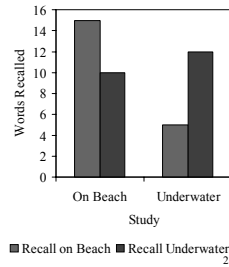
Additivity Definition of *Interaction*

- ⊞ An interaction occurs when the effects of one IV cannot simply be added to the effects of the other IV in order to predict how both treatments will simultaneously affect the DV
- ⊞ Determine what effect studying on the beach vs underwater has
- ⊞ Determine what effect recalling on the beach vs underwater has

22

Additivity Definition of *Interaction*

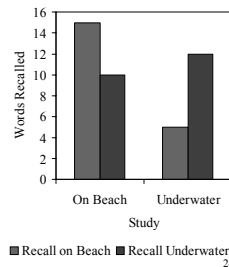
- ⊞ The average value of all conditions in which you study on the beach is $(15 + 10) / 2 = 12.5$
- ⊞ The average value of all conditions in which you study underwater is $(5 + 12) / 2 = 8.5$
- ⊞ So studying underwater leads to you recalling $12.5 - 8.5 = 4$ fewer words



23

Additivity Definition of *Interaction*

- ⊞ The average value of all conditions in which you recall on the beach is $(15 + 5) / 2 = 10$
- ⊞ The average value of all conditions in which you recall underwater is $(10 + 12) / 2 = 11$
- ⊞ So recalling underwater leads to you recalling $11 - 10 = 1$ more word



24

Additivity Definition of *Interaction*

- ⊕ If the IVs are independent of each other (i.e. they do *not* interact with other), then we should be able to predict the number of words recalled in three of the conditions given the number of words recalled in the fourth condition and the size of the main effects

25

Additivity Definition of *Interaction*

- ⊕ Number of words recalled when you study underwater and recall underwater = number of words recalled when you study on the beach and recall on the beach + effect of studying underwater + effect of recalling underwater
- ⊕ $\bar{X}_{\text{study underwater, recall underwater}} = \bar{X}_{\text{study on the beach, recall on the beach}} + \text{effect of studying underwater} + \text{effect of recalling underwater}$
- ⊕ $12 = 15 + (-4) + 1 = 12$

26

Additivity Definition of *Interaction*

- ⊕ Number of words recalled when you study underwater and recall on the beach = number of words recalled when you study on the beach and recall on the beach + effect of studying underwater
- ⊕ $\bar{X}_{\text{study underwater, recall on the beach}} = \bar{X}_{\text{study on the beach, recall on the beach}} + \text{effect of studying underwater}$
- ⊕ $5 \neq 15 + (-4)$
- ⊕ Because we cannot predict, the effects are not additive, and the variables interact

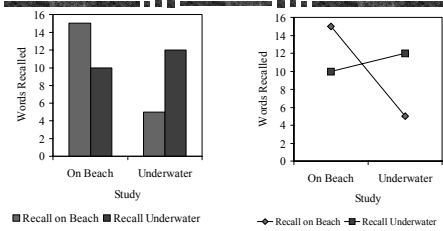
27

Additivity Definition of *Interaction*

- ⊕ Number of words recalled when you study on the beach and recall underwater = number of words recalled when you study on the beach and recall underwater + effect of recalling underwater
- ⊕ $\bar{X}_{\text{study on the beach, recall underwater}} = \bar{X}_{\text{study on the beach, recall on the beach}} + \text{effect of recalling underwater}$
- ⊕ $10 \neq 15 + 1$
- ⊕ Because we cannot predict, the effects are not additive, and the variables interact

28

Graphical Definition of Interaction



When the lines or bars on a graph are not parallel, then an interaction has occurred

29

Factorial ANOVA

- ⊞ The factorial analysis of variance answers each of the questions that can be asked
 - ⊞ Is there a main effect of the first IV?
 - ⊞ Is there a main effect of the second IV?
 - ⊞ Is there an interaction effect of the two IVs?
- ⊞ The ANOVA accomplishes these goals by giving us an F ratio for each of the questions that are asked

30

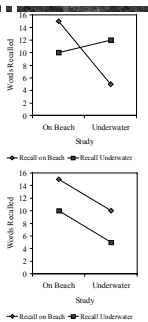
Factorial ANOVA

- ⊞ H_0 and H_1 for each of the main effects takes the same form as the single factor ANOVA H_0 and H_1 :
 - ⊞ $H_0: \mu_1 = \mu_2 = \dots = \mu_n$
 - ⊞ $H_1: \text{not } H_0$

31

Factorial ANOVA

- ⊞ H_0 and H_1 for the interaction take the following form:
 - ⊞ $H_0: \mu_{11} - \mu_{21} = \mu_{12} - \mu_{22}$
 - ⊞ $H_1: \mu_{11} - \mu_{21} \neq \mu_{12} - \mu_{22}$
- ⊞ μ_{12} = mean for level 1 of the first IV and level 2 of the second IV
- ⊞ μ_{21} = mean for level 2 of the first IV and level 1 of the second IV



32

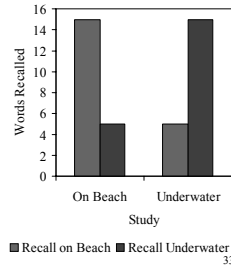
Factorial ANOVA

- Specify the alpha level

- $\alpha = .05$ for all tests

- Look at a graph and determine which main effects and interactions are likely

- No main effect of where you study
 - No main effect of where you recall
 - An interaction



Factorial ANOVA Summary Table

Source	SS	df	MS	F	p
IV1 (Study)	4	1	4	2	p=.07
IV2 (Recall)	2	1	2	1	p=.50
IV1 X IV2 (Study X Recall)	12	1	12	6	p=.02
Error (Within-groups)	88	44	2		
Total	106	47			

- Source = source of variance

- SS = sum of squares = top part of variance formula

- df = degrees of freedom = bottom of variance formula

- Main effect df = number of levels of IV - 1

- Interaction effect df = df of first IV X df of second IV

- Error df = $\Sigma(N_{\text{per condition}} - 1)$

- MS = mean squares = variance estimate

- F = F ratio = $MS_{\text{between-groups}} / MS_{\text{within-groups}}$

- p = probability value (significance)

Factorial ANOVA Decisions

Source	SS	df	MS	F	p
IV1 (Study)	4	1	4	2	p=.07
IV2 (Recall)	2	1	2	1	p=.50
IV1 X IV2 (Study X Recall)	12	1	12	6	p=.02
Error (Within-groups)	88	44	2		
Total	106	47			

- The line labeled with the name of the first IV tells you whether there is a main effect of that IV

- When the p value is less than or equal to α , then you can reject H_0 that all the means are equal

- That is, when $p \leq \alpha$, there is a main effect of the IV

Factorial ANOVA Decisions

Source	SS	df	MS	F	p
IV1 (Study)	4	1	4	2	p=.07
IV2 (Recall)	2	1	2	1	p=.50
IV1 X IV2 (Study X Recall)	12	1	12	6	p=.02
Error (Within-groups)	88	44	2		
Total	106	47			

- The line labeled with the name of the second IV tells you whether there is a main effect of that IV

- When the p value is less than or equal to α , then you can reject H_0 that all the means are equal

- That is, when $p \leq \alpha$, there is a main effect of the IV

Factorial ANOVA Decisions

Source	SS	df	MS	F	p
IV1 (Study)	4	1	4	2	p=.07
IV2 (Recall)	2	1	2	1	p=.50
IV1 X IV2 (Study X Recall)	12	1	12	6	p=.02
Error (Within-groups)	88	44	2		
Total	106	47			

- ⊞ The line labeled with the name of both of the IVs tells you whether there is an interaction of those IVs
 - ⊞ When the p value is less than or equal to α , then you can reject H_0
 - ⊞ That is, when $p \leq \alpha$, there is an interaction of the IVs

37

Writing the Results

- ⊞ The American Psychological Association has a precise format for writing the results of ANOVA in an article
- ⊞ $F(df_{\text{between-groups}}, df_{\text{within-groups}}) = F \text{ value}, p = p \text{ value}, MS_{\text{error}} = MS_{\text{within-groups}}, \alpha = \alpha \text{ level}$
- ⊞ The MS_{error} and α level are only with the first F ratio reported unless they change

38

Writing the Results

- ⊞ For the main effect of where you studied:
 - ⊞ $F(1, 44) = 2.00, p = .07, MS_{\text{error}} = 2.00, \alpha = .05$
- ⊞ For the main effect of where you recalled:
 - ⊞ $F(1, 44) = 1.00, p = .50$
- ⊞ For the interaction:
 - ⊞ $F(1, 44) = 6.00, p = .02$

39

Other ANOVAs

- ⊞ Just as in the t test and in the single factor ANOVA, there are different types of factorial design ANOVAs
- ⊞ If all the IVs are between-subjects then you should use a between-subjects ANOVA
- ⊞ If all the IVs are within-subjects then you should use a within-subjects ANOVA
- ⊞ If at least one IV is between-subjects and at least one IV is within-subjects, then you should use a mixed-design ANOVA

40

Other ANOVAs

- ⊕ The primary difference between the three types of factorial design ANOVAs is in the number of error (within-groups) estimates of variance there are
 - ⊕ Between-subjects ANOVA has 1 MS_{error}
 - ⊕ Mixed-design ANOVA has 2 MS_{errors}
 - ⊕ Within-subjects ANOVA has 3 MS_{errors}

41

Other ANOVAs

- ⊕ ANOVA is not limited to just 2 IVs -- you can have as many IVs as is necessary to answer your questions
 - ⊕ Because of the complexity of the design, you will rarely see more than 3 or 4 IVs used in a single experiment
- ⊕ When there are more than two IVs, you get information on main effects, simple interactions and higher-order interactions

42

Higher Order Interactions

- ⊕ A *higher-order interaction* occurs when the nature of the lower order interaction (such as the two variable interaction we have talked about) depends on the level of a third (or higher) IV
- ⊕ Higher order interactions are often difficult to interpret

43