ANOVA

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Multi-Level Experiments

- Often the research that psychologists perform has more conditions than just the control and experimental conditions
  - You might want to compare the effectiveness of two treatments to each other and to no treatment
- Such research designs are called *multi-level* because there are more than two levels to the independent variable

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Multiple t Tests

- How would you analyze data from a multi-level experiment?
- Given what we know so far, our only real option would be to perform multiple t-tests
- The problem with this approach to the data analysis is that you might have to perform many, many t-tests to test all possible combinations of the levels

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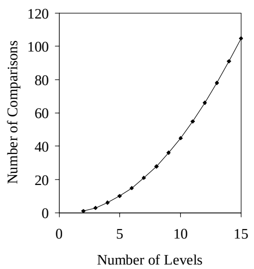
Multiple t-Tests

- As the number of levels (or conditions) increases, the number of comparisons needed increases more rapidly
- # comparisons = $(n^2 - n) / 2$
  - n = number of levels



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Why Not Perform Multiple t Tests?

- With a computer, it is easy, quick, and error-free to perform multiple t-tests, so why shouldn't we do it?
- The answer lies in α
  - α is the probability of making a Type-I error, or incorrectly rejecting H_0 when H_0 is true
- α applies to a single comparison
  - It is correctly called $\alpha_{\text{comparison wise}}$

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Why Not Perform Multiple t Tests?

- What happens to the probability of making at least 1 Type-I error as the number of comparisons increases?
- It increases too
- The probability of making at least one Type-I error across all of your inferential statistical tests is called $\alpha_{\text{familywise}}$ or α_{fw}

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α_{fw}

- If the probability of making a Type-I error on a given comparison is given by α, what is the probability of making at least 1 Type-I error when you make n comparisons?
- The simplest way to solve this problem is to realize that p(at least 1 Type-I error in n comparisons) + p(no Type-I errors in n comparisons) = 1
- So, α_{fw} = 1 - p(no Type-I errors)

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α_{fw}

- p(no Type-I errors in n comparisons) = p(no Type-I error on the 1st comparison) * p(no Type-I error on the 2nd comparison)* …* p(no Type-I error on the nth comparison)
- p(no Type-I errors in n comparisons) = p(no Type-I error on 1 comparison)^n
- p(no Type-I error on 1 comparison) = 1 - α

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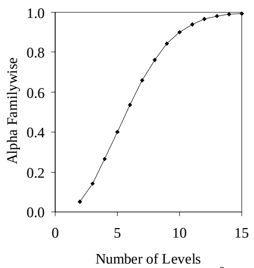
α_{fw}

- $\alpha_{fw} = 1 - (1 - \alpha)^n$
- As the number of comparisons increases, the probability of making at least 1 Type-I error increases rapidly



Why Not Multiple t-Tests?

- Performing multiple t-tests is bad because it increases the probability that you will make at least one Type-I error
- You can never determine which statistically significant results, if any, are probably due to chance

ANOVA

- Part of the solution to this problem rests in a statistical procedure known as the *analysis of variance* or *ANOVA* for short
- ANOVA replaces the multiple comparisons with a single *omnibus* null hypothesis
  - Omnibus -- covers all aspects
- In ANOVA, H_0 is always of the form:
  H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots. = \mu_n$
- That is, H_0 is that all the means are equal

ANOVA Alternative Hypothesis

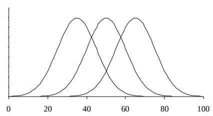
- Given the H_0 and H_1 must be both mutually exclusive and exhaustive, what is H_1 for ANOVA?
- Why isn't this H_1?
  H_1: $\mu_1 \neq \mu_2 \neq \mu_3 \neq \ldots. \neq \mu_n$
- In ANOVA, the alternative hypothesis is always of the form:
  H_1: not H_0

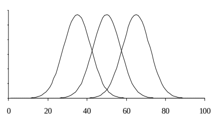
Two Estimates of Variance

- ANOVA compares two estimates of the variability in the data in order to determine if H_0 can be rejected:
  - Between-groups variance
  - Within-groups variance
- Do not confuse these terms with *between-subject* and *within-subjects designs*

Within-Groups Variance

- *Within-groups variance* is the weighted mean variability within each group or condition
- Which of the two distributions to the right has a larger within-groups variance? Why?



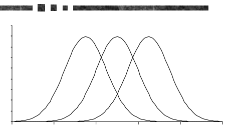


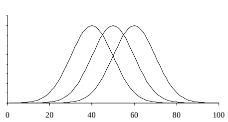
Within-Groups Variance

- What causes variability within a group?
  - Within-groups variation is caused by factors that we cannot or did not control in the experiment, such as individual differences between the participants
  - Do you want within-group variability to be large or small?
  - Within-groups variation should be as small as possible as it is a source of error

Between-Groups Variance

- *Between-groups variance* is a measure of how different the groups are from each other
- Which distribution has a greater between-groups variance?





Sources of Between-Groups Variance

- What causes, or is the source of, between-groups variance? That is, why are not all the groups identical to each other?
  - Between-groups variance is partially caused by the effect that the treatment has on the dependent variable
  - The larger the effect is, the more different the groups become and the larger between-groups variance will be

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Sources of Between-Groups Variance

- Between-groups variance also measures sampling error
  - Even if the treatment had no effect on the dependent variable, you still would not expect the distributions to be equal because samples rarely are identical to each other
  - That is, some of the between-groups variance is due to the fact that the groups are initially different due to random fluctuations (or errors) in our sampling

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Variance Summary

- Within-groups variance measures error
- Between-groups variance measures the effect of the treatment on the dependent variable and error

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F Ratio

- Fisher's F ratio is defined as:

$$F = \frac{\text{between} - \text{groups variance}}{\text{within} - \text{groups variance}}$$

$$F = \frac{\text{effect of treatment on DV variance} + \text{error variance}}{\text{error variance}}$$

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F Ratio

- What should F equal if H_0 is true?
  - When H_0 is true, then there is no effect of the treatment on the DV
  - Thus, when H_0 is true, we have error / error which should equal 1
- What should F be if H_0 is not true?
  - When H_0 is not true, then there is an effect of the treatment on the DV
  - Thus, when H_0 is not true, F should be larger than 1

How Big Does F Have to Be?

- Several factors influence how big F has to be in order to reject the null hypothesis
  - To reject H_0, the calculated F must be larger than the *critical F* which can be found in a table
  - Anything that makes the critical F value large will make it more difficult to reject H_0

Factors That Influence the Size of the Critical F

- α level -- as α decreases, the size of the critical F increases
- Sample size -- as the sample size increase, the degrees of freedom for the within-groups variance increases, and the size of the critical F decreases
- As the sample becomes larger, it becomes more representative of the population

Factors That Influence the Size of the Critical F

- Number of conditions -- as the number of conditions increase, so does the degrees of freedom for the between-groups variance term
- If the degrees of freedom for the denominator are larger than 2, then the size of the critical F will decrease as the number of conditions increases
- The opposite is true if the degrees of freedom for the denominator equal 1 or 2

Factors That Influence the Size of the Observed F

- Experimental Control -- as experimental control increases, within-groups variance decreases, and the observed F increases

ANOVA Summary Table

Source	SS	df	MS	F	p
Between-group	10	1	10	5	< .05
Within-group	20	10	2		
Total	30	11			

- SS = sum of squares -- the numerator of the variance formula = $\Sigma(X-\mu)^2$
- df = degrees of freedom -- the denominator of the variance formula
- df between-groups = # levels - 1
- df within-groups = Σ(# participants in a condition - 1)
- MS = SS / df
- F = $MS_{between\text{-}groups}$ / $MS_{within\text{-}groups}$
- p = p value (significance level)

ANOVA Assumptions

- ANOVA makes certain assumptions:
  - Sampling error is normal, or Gaussian in shape, and is centered around the mean of the distribution
  - Homogeneity of variance -- the variability within each group is approximately equal
  - Independence of observations

ANOVA Assumptions

- ANOVA is fairly robust (it will give good results even if the assumptions are violated) to the normality assumption and the homogeneity of variance assumption as long as:
  - the number of participants in each group is equal
  - the number of participants in each group is fairly large

Multiple Comparisons

- ANOVA only tells us if there is an effect
  - That is, are the means of the groups not all equal?
- ANOVA does not tell us which means are different from which other means
- Multiple comparisons are used to determine which means are probably different from which other means

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Multiple Comparisons

- Multiple comparisons are not fundamentally different from performing a t-test
- That is, both multiple comparisons and t-tests answer the same question: are two means different from each other
- The difference is that the multiple comparisons protect us from making a Type-I error even though we perform many such comparisons

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Multiple Comparisons

- There are many types of multiple comparison
  - E.g. Tukey, Newman-Keuls, Dunnett, Scheffé, Bonferroni
- There is little agreement about which particular one is best
- The different tests trade off statistical power for protection from Type-I errors

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Tukey Tests

- We will consider only the *Tukey test*; other people may feel that other tests are more appropriate
- The Tukey test offers reasonable protection from Type-I errors while maintaining reasonable statistical power

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Tukey Tests

- You should perform Tukey tests only when two criteria have been met:
  - There are more than two levels to the IV
  - The IV is statistically significant

Steps in Performing Tukey Tests

- Write the hypotheses
  - H_0: $\mu_1 = \mu_2$
  - H_1: $\mu_1 \neq \mu_2$
- Specify the α level
  - $\alpha = .05$

Steps in Performing Tukey Tests

- Calculate the *honestly significant difference (HSD)*

$$\text{HSD} = q_{\alpha,k,df_{\text{within-groups}}} \sqrt{\frac{MS_{\text{within-groups}}}{n}}$$

- $q_{\alpha,k,df\text{within-groups}}$ = tabled q value
- α = α level
- k = number of levels of the IV
- $df_{\text{within-groups}}$ = degrees of freedom for $MS_{\text{within-groups}}$
- $MS_{\text{within-groups}}$ = within-groups variance estimate
- n = number of participants

Steps in Performing Tukey Tests

- Take the difference of the means of the conditions you are comparing
- If the difference of the means is at least as large as the HSD, you can reject H_0
- Repeat for whatever other comparisons need to be made

A-Priori vs A-Posteriori Comparisons

- The Tukey comparisons we have been talking about are *post-hoc* or *a-posteriori* (*after the fact*) comparisons
  - That is, we did not specify prior to the experiment that we would perform these particular comparisons
- A-posteriori comparisons are the most frequently used comparisons in the social sciences

A-Priori vs A-Posteriori Comparisons

- There are other comparison techniques that are appropriate to use when, prior to the study, you have decided which comparisons you would like to make
- These comparisons are *a-priori* (*before the fact*) comparisons

A-Priori vs A-Posteriori Comparisons

- A-posteriori comparisons should only be performed after finding a statistically significant result in an ANOVA
  - This helps to reduce the probability of making a Type-I error
- A-priori comparisons can be, but need not be, preceded by an ANOVA
- A-priori tests tend to have greater statistical power than their a-posteriori counterparts

Between- vs Within-Subjects Designs

- Like the t-test, there are separate procedures to be used when you have between-subjects and when you have within-subjects designs
- Be sure to use the appropriate test
- SPSS (and many other statistical programs) call within-subjects ANOVA *repeated measures ANOVA*